Example: **- Divide $3x^2 - x^3 - 3x + 5$ by $x - 1 - x^2$, and verify the division algorithm.**

Given $p(x) = x^4 - 6x^3 - 26x^2 + 138x - 35$, $g(x)$ $x^2 - 4x + 1$

On dividing **p (x)** by **g(x)**, we get,

$$
\begin{array}{r|l}
 & x^2 - 2x - 35 \\
\hline
x^2 - 4x + 1 & x^4 - 6x^3 - 26x^2 + 138x - 35 \\
 & x^4 - 4x^3 + x^2 \\
 & - \quad + \quad - \\
\hline
 & -2x^3 - 27x^2 + 138x - 35 \\
 & -2x^3 + 8x^2 - 2x \\
 & + \quad - \quad + \\
\hline
 & -35x^2 + 140x - 35 \\
 & -35x^2 + 140x - 35 \\
 & + \quad - \quad + \\
\hline
 & 0 \\
\hline
\end{array}
$$

By Euclid's Division Algorithm

$P(x) = g(x) \times q(x) + r(x)$

$\therefore p(x) = (x^2 - 4x + 1)(x^2 - 2x - 35) + 0$

$= x^2(x^2 - 2x - 35) - 4x(x^2 - 2x - 35) + 1(x^2 - 2x - 35)$

$= x^4 - 2x^3 - 35x^2 - 4x^3 + 8x^2 + 140x + x^2 - 2x - 35$

$= x^4 - 6x^3 - 26x^2 + 138x - 35$

$= P(x)$ *Veryfied*

Solve the questions given below by the above method

Apply division algorithm to find the quotient q(x) and remainder r(x) on dividing p(x) by q(x) in each of the following and check these.

(a) $p(x) = x^3 - 6x^2 + 11x - 6$, $g(x) = x^2 + x + 1$

(b) $p(x) = 10x^4 + 17x^3 - 62x^2 + 30x - 3$, $g(x)$ $2x^2 + 7x + 1$

(c) $p(x) = 4x^3 + 8x + 8x^2 + 7$, $g(x) = 2x^2 - x + 1$

(d) $p(x) = 15x^3 - 20x^2 + 13x - 12$, $g(x) = 2 - 2x + x^2$

Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm

(a) $g(t) = t^2 - 3$; $p(t) = 2t^4 + 3t^3 - 2t^2 - 9t - 12$

Solution: Given, First polynomial = $t^2 - 3$

Second polynomial = $2t^4 + 3t^3 - 2t^2 - 9t - 12$

$$
\begin{array}{r l l l l l l}
 & & 2t^2 & +3t & +4 & & \\
\cline{3-7}
t^2-3 & \big) & 2t^4 & +3t^3 & -2t^2 & -9t & -12 \\
 & - & & & & & \\
 & & 2t^4 & +0t^3 & -6t^2 & & \\
\cline{2-7}
 & & & 3t^3 & +4t^2 & -9t & -12 \\
 & & - & & & & \\
 & & & 3t^3 & +0t^2 & -9t & \\
\cline{2-7}
 & & & & 4t^2 & +0t & -12 \\
 & & & - & & & \\
 & & & & 4t^2 & +0t & -12 \\
\cline{2-7}
 & & & & & & 0
\end{array}
$$

As we can see, the remainder is left as 0.

Hence we say that, t^2-3 is a factor of $2t^4+3t^3-2t^2-9t-12$

Check whether the first polynomial is a factor of the second polynomial by applying the division algorithm

(b)	**$g(t)=t^2-3$**	**$p(t)=2t^4+3t^3-2t^2-9t-12$**
(c)	**$g(x)=x^3-3x+1$**	**$p(x)=x^5-4x^3+x^2+3x+1$**
(d)	**$g(x)=2x^2-x+3$**	**$p(x)=6x^5-x^4+4x^3-5x^2-x-15$**